



INTERNATIONAL MATHEMATICS SUMMER CAMP IMSC23
MOCK TEST 2-COMBINATORICS

Date: Tuesday, 22nd June 2023 **Time:** 13:10-17:40
Number of problems: 3 **Total points:** 21

PROBLEMS

Problem 1. An international conference consists of 2023 representatives from each of 2023 different countries. Prove that 2023^2 people can be seated around a large round table such that if A and B are two distinct representatives from the same country, then the people sitting to the immediate left of A and to the immediate left of B are from different countries.

Problem 2. Two players Alex and Ben play the following game. At the start of the game, Alex chooses a positive integer n and n positive integers x_1, x_2, \dots, x_n . Alex keeps x_1, x_2, \dots, x_n secret, and truthfully tells n and $x_1 + x_2 + \dots + x_n$ to Ben. Ben now tries to obtain information about x_1, \dots, x_n by asking Alex questions as follows: each question consists of Ben specifying an arbitrary nonempty proper subset B of $\{1, 2, \dots, n\}$, and asking Alex for a subset A of $\{1, 2, \dots, n\}$ such that

$$A \neq B \text{ and } \sum_{i \in A} x_i = \sum_{j \in B} x_j.$$

After each question, if Alex fails to answer it truthfully, Ben wins the game immediately. After Ben asks as many questions as he wants, if Ben can specify the values of x_1, x_2, \dots, x_n , then Ben wins the game; otherwise Ben loses. Prove that Ben can guarantee a win.

Problem 3. In the land of Heptanomisma, four different coins and three different banknotes are used, and their denominations are seven different natural numbers. The smallest denomination of a banknote is greater than the sum of the denominations of the four different coins. A tourist has exactly one coin of each denomination and exactly one banknote of each denomination, but he cannot afford the book on numismatics he wishes to buy. However, the mathematically inclined shopkeeper offers to sell the book to the tourist at a price of his choosing, provided that he can pay this price in more than one way. (The tourist can pay a price in more than one way if there are two different subsets of his coins and notes, the denominations of which both add up to this price.) Find the natural number N such that (a) the tourist can purchase the book if the denomination of each banknote is smaller than N , and (b) the tourist may have to leave the shop empty-handed if the denomination of the largest banknote is N .