



INTERNATIONAL MATHEMATICS SUMMER CAMP IMSC23  
MOCK TEST 3-GEOMETRY

**Date:** Monday, 26th June 2023      **Time:** 13:10-17:40  
**Number of problems:** 3              **Total points:** 21

PROBLEMS

**Problem 1.** Let  $ABC$  be a triangle. The circle  $\omega_A$  through  $A$  is tangent to line  $BC$  at  $B$ . The circle  $\omega_C$  through  $C$  is tangent to line  $AB$  at  $B$ . Let  $D$  be the second point of intersection of  $\omega_A$  and  $\omega_C$ . Let  $M$  be the midpoint of  $BC$  and let  $E$  the intersection of  $MD$  and  $AC$ . Show that  $E$  lies on  $\omega_A$ .

**Problem 2.** Let  $k$  be the inscribed circle of non-isosceles triangle  $\triangle ABC$ , which center is  $I$ . Circle  $k$  touches sides  $BC, CA, AB$  in points  $P, Q, R$  respectively. Line  $QR$  intersects  $BC$  in point  $M$ . Let a circle which contains points  $B$  and  $C$  touch  $k$  in point  $N$ . Circumscribed circle of  $\triangle MNP$  intersects line  $AP$  in point  $L$ , different from  $P$ . Prove that points  $I, L$  and  $M$  are collinear.

**Problem 3.** Let  $ABC$  be a triangle with circumcircle  $\omega$  and  $\ell$  a line without common points with  $\omega$ . Denote by  $P$  the foot of the perpendicular from the center of  $\omega$  to  $\ell$ . The lines  $BC, CA, AB$  intersect  $\ell$  at the points  $X, Y, Z$  different from  $P$ . Prove that the circumcircles of the triangles  $AXP, B Y P$  and  $C Z P$  have a common point different from  $P$  or are mutually tangent at  $P$ .