INTERNATIONAL MATHEMATICS SUMMER CAMP IMSC23

MOCK TEST 4-NUMBER THEORY

Date: Wednesday, 28th June 2023 **Time**: 13:10-17:40
Number of problems: 3 **Total points**: 21

Problems

Problem 1. Find the number of integers c such that $-2023 \leq c \leq 2023$ and there exists an integer x such that $x^2 + c$ is a multiple of 2^{2023}

Problem 2. A positive integer m is perfect if the sum of all its positive divisors, 1 and m inclusive, is equal to $2m$. Determine the positive integers n such that $n^n + 1$ is a perfect number.

Problem 3. Let P be the set of all primes, and let M be a non-empty subset of P. Suppose that for any non-empty subset $p_1, p_2, ..., p_k$ of M, all prime factors of $p_1 p_2 ... p_k + 1$ are also in M. Prove that $M = P$